Strapdown Inertial Navigation System Requirements Imposed by Synthetic Aperture Radar

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The purpose of this paper is to demonstrate a means of specifying strapdown inertial navigation system (INS) requirements from synthetic aperture radar (SAR) requirements. The latter include allowable levels for quadratic and cubic phase shift, and side lobe levels [i.e., peak side lobe ratio (PSLR) and integrated side lobe ratio (ISLR)]. When these multiple considerations produce different INS requirements, of course, the tightest governs. Results obtained constitute a technique demonstration only, and do not represent any specific mechanization. In the process of this investigation several pitfalls in common procedures were identified; these are highlighted in the discussion. A brief background description is provided in the Appendix for those unfamiliar with the analysis of SAR degradations.

Nomenclature

$\tilde{A}$	= acceleration error, ft/s^2
$d;d$	= IMU-to-radar displacement in airframe coordinates; magnitude of d, ft
g	= acceleration due to gravity, ft/s^2
$\tilde{J}$	= jerk error, ft/s^3
j/H	= ratio of gyro inertia to angular momentum, s
j/P	= ratio of accelerometer inertia to pendulosity, ft
K^2	= number of error contributions to be rss
L	= static lift, g
l,m,n	= conventional airframe axes
N	= number of coherent integration intervals per SAR frame
n_ω	= total angular drift rate, rad/s
R_R, R_S	= position vector of the radar antenna phase center and strapdown IMU, respectively, ft
$\tilde{t}$	= timing uncertainty, s
t_c	= coherent integration interval, s
T_M	= mapping segment duration, s
T_W	= effective averaging interval of nav update Kalman filter, s
$[T]$	= orthogonal transformation from airframe to reference axes
$\tilde{v}$	= velocity vector error, ft/s
X,Y,Z	= inertial instrument axis directions
Δv	= quantization of integrating accelerometer, ft/s
$\delta\{\ \}$	= variation in $\{\ \}$
λ	= radar wavelength, ft
ρ	= navigation reference frame profile rate, rad/s
$\tilde{\phi}$	= motion compensation phase error, rad
ψ	= INS tilt, rad
1_R	= unit vector along direction of R_R
$(\dot{\ })$	= time derivative of ( )
$(\ )^T$	= transpose of ( )
$(\tilde{\ })$	= error in ( )
$\lvert\cdot\rvert$	= absolute magnitude of ($\cdot$)

I. Introduction

KNOWLEDGE of gimballed platform requirements for SAR mapping, while not universal, is becoming fairly widespread. When there is relative motion and/or large distance between the inertial navigation system (INS) and antenna, however, plans to use a strapdown mechanization must account for several error sources (particularly those involving rectification of motion-sensitive degradations) that would normally be ignored in a gimballed implementation. This paper assesses the impact for an extensive set of gyro and accelerometer errors applied to synthetic aperture radar (SAR).

Discussion of these topics would be incomplete without noting several shortcomings of presently accepted procedures. The following items are believed to be critical:

A) Strapdown inertial measuring unit (IMU) specifications quite often stress the usual long-term behavior (e.g., nm/h) while giving little attention to crucial short-term performance criteria. Explicit requirements for short-term stability of strapdown velocity and attitude errors should be included in applicable specifications. Dependable means for testing short-term performance in flight must be established.

B) Available vibration information is typically sketchy and limited to translational motion. Confidence in design would call for improvements in all of the following aspects of vibration data collection: a) specific attention to the sites (airframe stations) where sensors and instruments are located; b) addition of gyros (as well as accelerometers) near those sites for flight test; c) generalization of data processing to include coherence functions (in-phase and quadrature cross-axis correlation for all combinations of waveform histories,[1] rather than restriction to individual waveforms.

C) Inertial instrument biases are often treated as an overall resultant carried forward between flight phases having different dynamics. In marked contrast to that approach, recognition of separate contributors and the ramifications of their dependence on the dynamic environment provided much of the motivation for this paper.

In addition, the following items are worth noting:

D) Accelerometer bias states are often included, somewhat indiscriminately, in transfer alignment formulations. In many cases they cannot be separated from tilt effects; unobservable states should never be included in an estimation algorithm.

E) Typical gyro scale factor and/or mounting alignment accuracies are marginal for dynamics with large rotational excursions.

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F) On-line corrections of inertial instrument error sources, using laboratory-derived coefficients, are acknowledged and enthusiastically endorsed. However, there is no reason to believe that all coefficients age at the same rate. Since imperfections in these corrections of course constitute uncompensated degradations, more detailed information is needed about stability of lab calibration.

With the above items in mind, attention can be turned to conditions adopted for the analysis to be presented:

1) No autofocus operation is performed on the SAR data; all gate placement and phase adjustment commands must depend on onboard inertial sensors.

2) The strapdown IMU, consisting of three gyros and three accelerometers, is mounted on an antenna (gimballed or electronically steerable) structure.

3) A gimballed master INS is available for repeated in-flight leveling and azimuth alignment of the strapdown reference. The combination can also be externally updated from radar or other [e.g., Global Positioning System (GPS)] observations, over intervals denoted T_W. Between these update/alignment intervals, there are SAR mapping mission segments of duration T_M (in general containing several SAR frames) within which the strapdown IMU acts as an autonomous source of short-term nav data for gate placement and phase correction.

4) The dynamic conditions experienced during update/alignment segments can, in general, differ from those (both static and vibratory) present during mapping segments. Thus the ability to "learn" inertial instrument bias effects in transfer alignment, and apply that knowledge as in-flight calibration data during SAR operation, can be exploited only to a very limited extent. Further elaboration of this point appears at appropriate points in the subsequent text.

5) Each individual strapdown error source is allowed to reach $(1/K)$ of the total error allowed (so that as many as K^2 separate sources of degradation could be rss without exceeding that total).

6) It is recognized that several inertial instrument errors are compensated in the front-end processing; only the uncompensated residual error is being addressed here.

7) With standard (l,m,n) aircraft axis conventions, the Z axes for inertial instruments 1, 2, and 3 under nominal reference conditions will point along (m,l,m), respectively. Thus the lift force will not appear along any inertial instrument Z axis for electronically steerable arrays nor, under conditions of no sightline depression, for gimballed antennas.

8) SAR motion compensation accuracy requirements are first derived for straight-and-level flight conditions during the mapping segment, and generalization to maneuvering flight is deferred to Sec. IV.

9) Strapdown requirements are based on SAR mapping alone, without regard to other radar modes or operations that may accompany it. In combination with condition 8 above, it follows that azimuth alignment is not critical in the immediate development. Again, the subsequent consideration of SAR mapping during maneuvers can address this issue separately, and the perspective here will be clearer as a result.

Since the system to be analyzed in hypothetical, nominal parameter values (1 ft, 1 g, 1 Hz, 1 deg) can be assumed, e.g.,

1) There is one coherent integration time interval per SAR frame.

2) Lever arm separation between IMU and radar antenna is 1 ft.

3) Mean product of specific force components is characterized by superposition of the following two effects:

a) (1-g rams) × (1-g rms) = 1 g^2 tightly coupled translational vibration, and

b) (deterministic 1-g lift) × (0.1-g horizontal component) = 0.1 g^2 for cruise.

4) Mean product of angular rates is dominated by narrowband random components, each in conformance to a 1-deg amplitude oscillation at 1-Hz center frequency. This produces rms angular rates of approximately 0.08 rad/s per axis and, with tight coupling, essentially 0.006 (rad/s)^2 for angular rate-squared sensitive errors.

5) Wavelength $\lambda = 0.1$ ft.

6) Coherent integration time $t_c = 2$ s.

7) The maximum duration of the mapping segment T_M is 100 s.

8) The value of K (from condition 5 of Sec. I) is 4, so that each separate contributor to the error budget can be one-quarter of the allowable total.

II. Background

Instantaneous position vectors of the radar antenna phase center and the strapdown IMU will be denoted here as $\boldsymbol{R}_R$ and $\boldsymbol{R}_S$, respectively, expressed in a reference (e.g., locally level) coordinate frame with the origin at a designated point in the mapped area. If $\boldsymbol{d}$ and $[T]$ denote the IMU-to-radar displacement vector in airframe coordinates and the orthogonal transformation from airframe to reference axes, respectively then

$$\boldsymbol{R}_R = \boldsymbol{R}_S + [T]\boldsymbol{d} \tag{1}$$

so that the motion compensation phase error is

$$\tilde{\phi} = 4\pi\delta\{\,|\boldsymbol{R}_R|\,\}/\lambda = (4\pi/\lambda)\delta\{(\boldsymbol{R}_R^T\boldsymbol{R}_R)^{1/2}\}$$
$$= (4\pi/\lambda)(1/2)(1/|\boldsymbol{R}_R|)\delta\{\boldsymbol{R}_R^T\boldsymbol{R}_R\} \tag{2}$$

which, to first order, is simply

$$\tilde{\phi} = (4\pi/\lambda)(\boldsymbol{1}_R^T\tilde{\boldsymbol{R}}_S + \boldsymbol{1}_R^T[\tilde{T}]\boldsymbol{d} + \boldsymbol{1}_R^T[T]\tilde{\boldsymbol{d}}) \tag{3}$$

where the variational operator $\delta\{\ \}$ is now replaced by the error notation $(\tilde{\ })$ and $\boldsymbol{1}_R$ denotes the unit range vector $\boldsymbol{R}_R/|\boldsymbol{R}_R|$. The bracketed factor on the right contains three terms which, in sequence, represent along-range components of the following effects.

1) Incremental† IMU position error appearing during a SAR frame. This includes effects of velocity quantization, acceleration error (tilt plus accelerometer errors plus vertical deflections), and contribution of gyro drift to cumulative position uncertainty.

2) Instantaneous attitude error. This includes the interaction between rapid rotations and attitude data timing offsets as well as quantization.

3) IMU/Radar displacement error. This includes unknown vibrations and phase center wander, plus interactions between rotational dynamics and static displacement uncertainty.

As an illustrative example, consider a constant along-range acceleration bias $\tilde{A}$, producing a contribution to the first term in the preceding equation,

$$\tilde{\phi} = (4\pi/\lambda)(1/2)\tilde{A}(t_c/2)^2 \tag{4}$$

$$(4\pi/\lambda)\tilde{A}(t_c/2)^2/2 < \pi/2 \tag{5}$$

when the number N of coherent integration intervals per SAR frame is unity; otherwise this limit must be divided by $\sqrt{N}$.

A requirement of this type actually dictates the need for both: 1) an initial tilt ceiling not to exceed $(1/K)$ $(\tilde{A}/g)$ rad, which really places demands on a master source for transfer alignment and/or aiding signals, and 2) the ability of the IMU

†Only the dynamic variations in rf phasing are critical for SAR motion compensation. Thus, while static position uncertainties can far exceed the wavelength, changes in those errors must be held to a small fraction of a wavelength during a SAR frame (typically a few seconds). Reference 2 contains a brief analysis on pp. 260-266.

when the bias is integrated over $(t_c/2)$. The rationale for this is that, with the phase reference set in correspondence to the center of the coherent integration interval, the phase error has a duration of half this interval to accumulate. A common limit of acceptability for this effect is one-quarter cycle of cumulative phase error; thus

$$(4\pi/\lambda)\tilde{A}(t_c/2)^2/2<\pi/2 \tag{5}$$

when the number N of coherent integration intervals per SAR frame is unity; otherwise this limit must be divided by $\sqrt{N}$.

A requirement of this type actually dictates the need for both: 1) an initial tilt ceiling not to exceed $(1/K)$ $(\tilde{A}/g)$ rad, which really places demands on a master source for transfer alignment and/or aiding signals, and 2) the ability of the IMU to hold an initial alignment within tighter tolerances, not just over the long-term average, but through any interval of mode-duration length (i.e., larger error sequences of alternating sign cannot be averaged).

It is condition 2 that imposes frequently overlooked requirements on the strapdown IMU and, excluding quantization effects (covered in Secs. III.C and III.E), short-term stability requirements for output errors should become standard specification items for IMU's to be used for such applications.

III. Analysis for Straight-and-Level Mapping Segment

In cruising flight, the contribution of a leveling error (ψ rad) to total horizontal acceleration error is simply the product $g\psi$. Thus since, on the basis of the preceding equation, maximum acceptable total horizontal error is

$$\tilde{A}=\lambda/t_c^2 \tag{6}$$

and since the tilt effect is only one of K^2 contributors (to be rss with all accelerometer biases and vertical deflection effects), the allowable tilt is

$$\psi=(1/K)(\tilde{A}/g)=\lambda/(Kgt_c^2) \tag{7}$$

It is reiterated here that, due to condition 4 of Sec. I, any correlation that may have existed initially between ψ and total accelerometer bias cannot be assumed to stay maintained. Since the initial correlation is negative (i.e., tilt and accelerometer bias effects counteract during initial alignment) the rss operation used here is conservative when much of the initial correlation remains. When this is not the case, however, a potential pitfall is present in this approach. Since the resulting problem is most likely to arise under maneuvering conditions, and since the full scope of that problem includes material yet to be covered, the subject will be discussed in Sec. IV.

A. Influence of Navigation Update/Alignment Phase

At this point a value must be established for the duration T_W of the data window for measurements used to achieve the necessary strapdown IMU leveling accuracy. This quantity, controlled by the spectral density of process noise in a Kalman filter (see, Ref. 2, Chap. 5), represents a nominal interval over which these measurements are effectively averaged. A cardinal rule for realization of expected performance in a Kalman filter is

$$\text{Duration of model fidelity} > T_w > \text{Time required for observability} \tag{8}$$

The first decision made along these lines was to abandon all hope of identifying each separate contributor to gyro and accelerometer error budgets (Secs. III.B-D to follow) as augmenting states for in-flight calibration. To do that would have required a state dimensionality of impractial size for operational software and, of greater importance, an impracticable sequence of aerodynamic maneuvers needed to isolate each augmenting state unambiguously. Thus the model used to represent flight dynamics in this application contains only time-invariant misorientation angles in addition to three-dimensional position and velocity states. For the goal of achieving strapdown leveling accuracy there are three different conditions to be addressed, i.e., 1) master INS update (Sec. III.A.1), 2) transfer of master INS leveling accuracy to the strapdown slave (Sec. III.A.2), and 3) full transfer alignment, including azimuth axis (Sec. IV).

1. Initialization

On occasions when master INS leveling accuracy is poorer than that needed for the slave, nav update from external aids (e.g., radar, GPS) will clearly be the requisite operation. The position/velocity/angular misorientation state selection just described then calls for the Kalman filter formulation defined in Ref. 2, Sec. 6.5. An upper limit for model fidelity duration in Eq. (8) is therefore one-tenth of the Schuler period, or roughly 500 s. The other side of this inequality is influenced by the navaid accuracy in the expression

$$(\text{Navaid error tolerance})/(0.5\, gT_W^2)<\psi \tag{9}$$

that is, while considerable flexibility exists for update scheduling, there must be enough aiding information collected in each interval of duration T_W to deduce tilt to within ψ rad. Typically a satisfactory result can be obtained from differential GPS measurements, with accuracies described in Ref. 3, averaged over a data window of a few hundred seconds. At the same time, gyros must not produce enough integrated drift effect to add another ψ rad of tilt within the same period. Therefore, allowable total drift (n_ω) is subject to the restriction

$$n_\omega<\psi\text{rad}/T_W\text{sec} \tag{10}$$

2. Leveling Alignment Transfer

When the master INS is level to within ψ rad but the strapdown is not, the value of T_W can be reduced to substantially less than 1 min.[4] A value of $T_M=100$ s would then allow reasonable efficient utilization ($T_W \ll T_M$ for low fractional dead time between mapping segments). The above inequality would then be easily satisfied if

$$n_\omega=\psi/T_M \tag{11}$$

and, since each individual gyro drift source is again one of K^2 separate contributors, the allowable limit for each [in view of Eq. (7)] is

$$\psi/(KT_M)=\lambda/(K^2gt_c^2T_M) \tag{12}$$

or about 0.5 μrad/s $\doteq$ 0.1 deg/h. For thoroughness, it is noted that total effective drift about a level axis in straight-and-level flight [Ref. 2, Eqs. (3-39) and (3-46)] includes the product (azimuth misalignment expressed in rad) × (angular rate ρ of the nav reference coordinate frame). Thus Eq. (12) also imposes the following upper limit on allowable azimuth misalignment:

$$(1/\rho)\psi/(KT_M)=\lambda(K^2g\rho t_c^2T_M) \tag{13}$$

In practice this restriction may be superseded by requirements of a separate operation or mode other than SAR.

B. Accelerometer Requirements Derived from Quadratic Phase Shift

Quadratic and cubic phase terms arise from quasistatic components of acceleration and gyro errors, respectively. Taken directly, these terms produce a power series in time; since this is not an expansion in orthogonal functions, the overall error history is sometimes re-expressed in terms of a Legendre series. However, that procedure will not be followed here since retention of the original power series produces a slightly conservative requirements specification.

With parameter values from Sec. I substituted into Eq. (6), allowable total horizontal acceleration bias is

$$\tilde{A} = 0.1 \div 2^2 = 0.025 \text{ ft/s}^2 = 0.0008\, g \tag{14}$$

which, with $K=4$ from Sec. I, imposes a limit of 200 μg for each separate source of acceleration bias (including effects of verticality error as in the present development). Following is a derivation of specifications for each contributor to the integrating accelerometer error budget. The approach uses methods that are fairly well documented (e.g., Ref. 5 and Ref. 2, Chap. 4).

Null Bias. Immediately from above, allowable null bias is 0.0002 g.

Scale Factor. In straight-and-level flight, accelerometer scale factor error does not produce any static horizontal acceleration bias.

Misalignment. It is fairly well recognized (see, for example, Ref. 2, pp. 72 and 111) that accelerometer alignment is not at all critical for credibility of the inertial nav data per se. Departure from an intended aerodynamic reference is not critical, and even input axis (IA) nonorthogonality can be "dumped" into the altitude channel (which causes it to be largely ineffective). On this basis the allowable misalignment of an accelerometer about the intended direction of each orthogonal axis [output axis (OA) and pendulous axis (PA)] should be dictated by requirements of whatever other system operations accompany SAR mapping.

OA Sensitivity. The coefficient (denoted j/P) for sensitivity to angular acceleration about the OA, multiplied by the 0.006 $(\text{rad/s})^2$ established in Sec. I, must not exceed 0.0002 g. Thus (j/P) can be 0.033 g per rad/s^2 at most.

It is worth noting that, although the instantaneous degradation from this phenomenon is proportional to angular acceleration, the rectified bias is proportional to mean squared angular rate. Reference 2, Chap. 4, describes this and other rectification effects.

Anisoinertia. The aforementioned 0.006 $(\text{rad/s})^2$ figure, multiplied by the anisoinertia coefficient, cannot exceed 0.0002 g. Maximum allowed anisoinertia coefficient is then 0.033 $g/(\text{rad/s})^2$.

Rebalancing Loop Response Delay. Bandwidth of presently available strapdown accelerometers easily satisfies any conceivable motion spectrum of interest. For completeness another requirement must be imposed on accelerometer output processing, due to unequal bandwidths of gyro and accelerometer loops. With no compensation, velocity increments would be transformed through previous, rather than current, gyro-fed attitude matrices. Velocity increments must therefore be delayed to counteract this timing offset and, furthermore, each accelerometer output should be time-trimmed to equalize these delays as closely as possible. Any imperfection in this timing adjustment is charged against the overall timing mismatch budget as discussed below.

Accelerometer Output Delay Mismatch. The aforementioned timing mismatch cannot exceed the ratio of (0.0002 g)/(mean product of vibratory angular rate × the translational acceleration along an orthogonal axis). Motion specifications for correlations between translational and rotational vibration components are quite scarce but, even with tight coupling assumed between the 1-g and the 0.08 rad/s (from Sec. I), allowable mismatch is essentially 3 ms. With currently available accelerometer loop bandwidths, this requirement is so lenient that it hardly needs to be stated.

Vibropendulosity. This coefficient multiplied by 1.1-g^2 [parameters 3a and 3b in Sec. I] cannot exceed 0.0002 g; thus, vibropendulosity $<0.00019\ g/g^2$.

Size Effect. Distance between accelerometers must not exceed the ratio $(0.0002\ g)/[0.006\ (\text{rad/s})^2]$, or $0.033 \times 32.2 =$ approximately 1 ft; this is quite lenient, since strapdown packages have separations measured in inches, not feet.

Residual Sculling. With finite inertial instrument resolution and data rates, some sculling effects remain at the output; in the presence of motion levels already discussed, the residual bias from that source cannot exceed 0.0002 g.

The foregoing analysis addresses an extensive, but not exhaustive, list of accelerometer parameters. One additional effect, quantization, allows portions of the motion experienced to go temporarily unnoticed (i.e., until the next quantum threshold is crossed). The next section addresses this phenomenon.

C. Velocity Quantization

One straightforward way to characterize this degradation is to visualize a steady specific force (A) producing a velocity ramp with a slope of (A) ft/s/s while the apparent velocity history would be a staircase function just below, and tangent to, the ramp. Velocity error would then be a sawtooth function with frequency f and amplitude equal to the integrating accelerometer resolution (Δv); furthermore,

$$f = A/\Delta V \tag{15}$$

and the amplitude of this spectral component in the sawtooth is $(\Delta v/\pi)$. In accordance with the velocity error expressions (see Ref. 2, p. 264), this produces a normalized error of order

$$(\Delta v/\pi)/(f\lambda) = (\Delta v)^2/(\pi\lambda A) \tag{16}$$

A value of 0.032 for this quantity produces $[20 \log_{10}(0.032)] \doteq -30$ dB PSLR. At the 0.1-ft wavelength with $A > 0.01\ g$, this implies a quantization of 0.01 ft/s. It is acknowledged that a specific force above the allowable level of each bias contributor (0.0002 g in the current example) but below the quantization level, held steady to within 0.0002 g during t_c at that low level, would interact with this quantization to produce unacceptable quadratic phase shift. These conditions are unlikely to hold, however, even in cruise. Velocity quantization is best characterized as an error source that distributes itself among a wide span of SAR image cells, and thus fails to produce the severe degradation that would have resulted from concentration within a narrow spectral region.

D. Gyro Drift

Cubic phase shift arises from a rate of change of acceleration error,

$$\tilde{J} = g n_\omega \tag{17}$$

triply integrated over each half of the coherent integration interval [recall the discussion following Eq. (4)]. For a maximum allowable shift of $\pi/8$ rad,

$$(4\pi/\lambda)(1/6) g n_\omega (t_c/2)^3 < \pi/8 \tag{18}$$

or

$$n_\omega < 3\lambda/(2 g t_c^3) \tag{19}$$

which is superseded by Eq. (12) since $K^2 T_M \gg t_c$. Gyro coefficients will now be determined by the methods used in Sec. III.B.

Null Bias. With the null defined as the motion-insensitive component of drift bias, calibration procedures can reduce this effect to about the dynamic drift level, and a steady bias is not critical. A practical specification might reasonably call for reduction of this effect by an order of magnitude; null bias can then be 10 times the allowable active level, or 1 deg/h.

Turn-on Bias Variability. Allowable variation is 0.1 deg/h, from Eq. (12).

Scale Factor and Misalignment. Both of these effects are inactive in straight-and-level flight. Their consideration is thus deferred to Sec. IV.

OA Sensitivity. The coefficient (denoted j/H) for gyro sensitivity to angular acceleration about the OA, multiplied by the 0.006 $(rad/s)^2$ [parameter 4 of Sec. I], must not exceed 0.1 deg/h, or approximately 0.5 μrad/s. Thus (j/H) can be 0.00008 s at most.

Anisoinertia. The aforementioned 0.006 $(rad/s)^2$ figure, multiplied by the anisoinertia coefficient, cannot exceed the same 0.5 μrad/s figure. Maximum allowable anisoinertia coefficient is then 0.00008 s.

Rebalancing Loop Response. Since nothing above a few Hz rotational frequency was postulated in the assumed motion (Sec. I), typical bandwidths of available gyros do not violate any requirements here.

Gyro Output Delay Mismatch. The previous established figure of 0.006 $(rad/s)^2$, multiplied by gyro timing mismatch, cannot exceed 0.5 μrad/s. Allowable time lag mismatch is then also 0.00008 s.

g-Sensitive Drift. With 1-g nominal sustained lift, allowable g-sensitive drift is simply 0.1 deg/h/g. This applies to both axes (X and Z).

g-Squared sensitive Drift. With vibratory translation characterized by item 3a of the parameter list in Sec. I, allowable g-squared sensitive drift is of order 0.1 deg/h/g^2. This applies to both anisoelastic (in-phase correlated) and cylindrical (quadrature correlated) components.

Residual Commutation Error. With finite gyro resolution and computational rates, some coning effects can remain at the output; in the presence of motion levels described in Sec. I, residual drift bias from that source cannot exceed 0.1 deg/h.

E. Angular Increment Data

In this application, PSLR and ISLR are essentially determined by velocity quantization and lever arm vector uncertainty. At typical gyro quantization levels on the order of a few arc-sec, lever arm vector uncertainty is dominated by timing effects. With the 0.08 rad/s rms angular rate, rms angular uncertainty θ accumulated in $\tilde{t}$ s is 0.08 $\tilde{t}$ rad and, for a 1-ft separation (d) from sensor package to antenna, PSLR corresponding to a sinusoidal error at a frequency denoted F would be determined from the ratio

$$\tilde{v}/(F\lambda) = 2\pi F\theta d/(F\lambda) = 2\pi(0.08\tilde{t})(1\text{ft})/0.1\text{ ft} = 5\tilde{t} \tag{20}$$

ISLR in this case is essentially twice the above level, or 10 $\tilde{t}$. At $\tilde{t}=0.00058$ [corresponding to $(1/\sqrt{12})\times(1/500)$ s at 500 Hz data rate for attitude information] these figures correspond to [20 $\log_{10}(0.0029)$] and [20 $\log_{10}$ (0.0058)] or roughly −50 and −44 dB, respectively; thus the 30 dB figure derived in Sec. III.C effectively governs the side lobe levels at 0.01 ft/s quantization. With finer velocity quantization the system could approach the 50 and 44 dB levels just derived for the data rate used here. Actually these figures depend heavily on methods used to reprocess the short-term attitude history and on detailed assumptions regarding error waveforms (which influence the ratio of rms to peak amplitudes, error statistics, etc.); they should thus be regarded as approximations subject to a few dB variation.

F. Summary of Strapdown Requirements

Results obtained for the example just cited are summarized in Table 1. They are restrictive in scope as already explained; extension to more complex scenarios is discussed in the next section.

These figures should be regarded as nominal values, subject to adjustment as appropriate for design needs. There are, for example, factors to be applied such as the square root of the number of partially overlapping coherent integration intervals in a SAR frame. Also, some error contributors may exceed the nominal level while others are reduced, as long as the overall rss does not exceed the allowable total. In any case, the methodology is available to coordinate a complete balanced design in accordance with any set of conditions and system requirements.

IV. Operation in the Presence of Acceleration

For aircraft experiencing speed changes, pullups, or turns, static specific force is no longer restricted to the vertical direction. The basic relation governing propagation of velocity error $\tilde{v}$ remains, however,

$$\dot{\tilde{v}} = \psi \times (\text{specific force}) + (\text{accelerometer error}) \tag{21}$$

as resolved along locally level reference coordinates, for strapdown as well as gimballed platform (Ref. 2, Chap. 4). In this application, however, the azimuth reference of interest is vehicle-based. Unlike the straight-and-level case (wherein the constant verticality of the lift force prompts a subdivision of orientation error per Table 1 footnote), azimuth misalignment here is defined only in terms of overall uncertainty in relative orientation between radar beam and acceleration vector. This lends immediate clarification to the subject under consideration, e.g., at a nominal 45-deg bank the azimuth misalignment now produces time-varying velocity and position errors similar to those associated with leveling error effects. Other specific force values can be analyzed by g scaling. These generalized conditions are by turns beneficial and detrimental, i.e.,

1) Horizontal acceleration components considerably enhance observability of azimuth misorientation during the update/alignment phase.[4,6]

2) Equation (13) no longer governs allowable azimuth misalignment if these more dominating effects are active during the mapping segment; allowable radar/acceleration vector misalignment uncertainty becomes simply (static acceleration error allowed by quadratic phase shift)/(static acceleration).

3) Differences in static conditions often produce changes in vibratory motions that influence the strapdown inertial instrument errors.

Table 1 Tabulation of inertial instrument error coefficients (level flight)

Parameter	Gyro	Accelerometer
Null bias	1 deg/h	0.0002 g
Turn-on bias variability	0.1 deg/h	N/A
Scale factor	See Sec. IV	See Sec. IV
Y-axis misalignment	See Sec. IV	Not critical[a]
Z-axis misalignment	See Sec. IV	Not critical[a]
OA sensitivity	0.00008 s	0.033 g-s^2
Anisoinertia	0.00008 s	0.033 g-s^2
Rebalancing loop response lag	Not critical	Not critical
Differential response lag	0.00008 s	Not critical
X-axis g-sensitive drift	0.1 deg/h/g	N/A
Z-axis g-sensitive drift	0.1 deg/h/g	N/A
g^2-sensitive drift	0.1 deg/h/g^2	N/A
Residual commutation error	0.1 deg/h	N/A
Residual sculling	N/A	0.0002 g
Vibropendulosity	N/A	0.0002 g/g^2
Size effect	N/A	Not critical
Velocity quantization		0.01 ft/s
Data rate		500 Hz

NOTE: X and Y axes denote input and output axes, respectively; Z axis denotes SRA for gyros and pendulous axis for accelerometers. Values in this table are derived from the hypothetical conditions enumerated in Sec. I.

[a] While accelerometer alignment is not critical for navigation in straight-and-level flight, there are other requirements (e.g., alignment between IMU and radar) that could be expressed as retrictions on allowable uncertainty in the accelerometer-based coordinate frame. In the present context, however, that frame is coincident with aircraft coordinate axes by definition, so that the total error in angular displacement between lift vector and radar beam subdivides, for straight-and-level flight, into 1) INS leveling error and 2) uncertainty in relative orientation between radar and IMU.

4) Three degradations that can be ignored in straight-and-level flight become active. The following conditions are now added to inertial instrument requirements.

Accelelerometer Scale Factor. In the presence of a nominal static lift of Lg, this effect must not introduce more than ψLg error during transfer alignment, since this would not be readily distinguishable from a tilt of ψ rad in a subsequent level mapping phase. If g levels experienced during mapping deviate significantly from unity, this requirement could also be scaled accordingly. Preferably, however, the analysis should be supported by further steps guided by considerations presented at the end of this section.

Gyro Scale Factor. The maximum angular orientation change to be experienced, multiplied by the scale factor error, must not exceed ψ rad (which would produce ψg in nominally level flight). Again the considerations at the end of this section are cited for further analysis.

Gyro Misalignments. These are defined as departures of IA direction (or, when skewed axes are used for redundancy management, *uncertainties* in these departures) from the accelerometer-based coordinate frame established in Ref. 2, pp. 72 and 111. Although the direction of this effect differs from that of gyro scale factor error, the magnitude is computed in the same way; allowable misalignment about each axis [OA and spin reference axis (SRA)] is on the order of (allowable attitude error)/(rotational excursion).

Attitude changes also influence the third term of Eq. (3) directly, and the second term indirectly. As with the other effects described here, quantitative results depend upon applicable scenario dynamics.

Material just presented only highlights the issues to be addressed with general motion sequences. Reasons for this are traceable to variations in correlation between initial errors at the start of a mapping segment (e.g., immediately after transfer alignment) and errors present during that segment. It is quite well known, for example, that achievable verticality accuracy during in-flight leveling is inextricably linked to accelerometer bias effects. Typically what is obtained is a combined effect of total instrument biases algebraically summed with the interaction between attitude error and prevailing forces [one needs to look no further than Eq. (21) for an explanation]. There are several important ramifications:

1) The common practice of introducing accelerometer bias states into transfer alignment algorithms is chancy at best and flatly inappropriate for rapid leveling. Total accelerometer bias is not separately observable under many realistic flight conditions.

2) The pitfall mentioned after Eq. (7) can be demonstrated as follows: A 1-mrad tilt, counteracted by 800 μrad resultant accelerometer bias, would produce an apparent 200 μg offset in a leveling operation. If instrument bias effects subsequently changed sign while maintaining the 800 μg total magnitude, SAR performance could be unacceptable (due to more than twice the allowable amount of quadratic phase shift in the example of Sec. III.B). This is only one simple example illustrating how short-term IMU error correlation dynamics can make or break a planned sequence of in-flight operations. In this example it would be prudent to tighten all error coefficient specifications by a factor of 2 when loss of correlation (Sec. I, condition 4) appears likely; more generally a complex scenario calls for either consistently conservative design procedures or rigorous simulation.

V. Conclusions

An approach has been demonstrated for establishing strapdown IMU requirements to meet the needs of synthetic aperture radar. Common deficiencies in strapdown IMU specifications as normally stated are identified and analyzed. Quantitative results do not represent any existing system, and are not intended to (some may, in fact, look strange); nevertheless the procedure followed herein provides an illustrative rationale to be used. Results obtained in any application will depend heavily upon coherent integration time, duration of update/alignment phase and mapping segment, and applicable dynamics. Of these, the vibration environments (including rotational effects with cross-axis correlations, both in-phase and quadrature) are generally the least available. Absence of this information will force designers to make assumptions, with the inevitable risks of degraded performance vs overdesign.

Appendix: Background for SAR Degradation Analysis

It was deemed appropriate to provide, within constraints of brevity, the following descriptive material for those unfamiliar with the topic. Concepts presented here do not rigorously conform to mechanization, but are simplified to demonstrate only susceptibility to degradations. For that purpose, SAR imaging is characterized as the determination of relative amplitudes for radar reflections from each range/Doppler cell (analogous to a TV picture element) in a swath. Geometric relations that define intersections of range and Doppler loci are combined with the collection of amplitude information, which may be obtained from magnitudes of complex numbers C_{nk} as follows: After centering to maintain the origin of a frame at a fixed range/Doppler cell index designation, the response history of the nth range gate to a sequence of transmitted radar pulses may be expressed as

$$G_n(t) = \sum_k C_{nk} \exp\{-i\omega_k t\}, \qquad 0 \le t \le t_c \tag{A1}$$

where $i = \sqrt{-1}$ and ω_k denotes the kth Doppler frequency in rad/s. The contents of this gate can then be decomposed into spectral components through repeated operations with *correlating functions* of the form

$$E_j(t) = \exp\{+i(\omega_j t + \tilde{\phi})\} \tag{A2}$$

where $\tilde{\phi}$ represents the phase error from Eq. (2) which, if zero, would have produced just the complex amplitude C_{nk} when $j=k$ in the correlating operation

$$\frac{1}{t_c}\int_0^{t_c} G_n(t) E_j(t) \mathrm{d}t \tag{A3}$$

In general, however, correlator outputs include extraneous information exemplified thus: Suppose a velocity error history $\tilde{v}(t)$ produces a range error history in accordance with

$$\delta |\boldsymbol{R}_R| = \int v(t) \mathrm{d}t \tag{A4}$$

and, if

$$\tilde{v}(t) = \tilde{v}_0 \sin\omega_0 t \tag{A5}$$

then, from Eqs. (2), (A4), and (A5),

$$\tilde{\phi} = -\left[\frac{4\pi\tilde{v}_0}{\lambda\omega_0}\right]\cos\omega_0 t \tag{A6}$$

When this amplitude is $\ll 1$, then Eq. (A2) reduces to

$$E_j(t) \doteq \left[1 - i\frac{4\pi\tilde{v}_0}{\lambda\omega_0}\cos\omega_0 t\right]\exp(i\omega_j t) \tag{A7}$$

and, superimposed on the desired correlator output, there will be attenuated responses (consistent with the sideband amplitude just established) from reflectors in the nth range gate but displaced in Doppler by ω_0 rad/s above and below the appropriate frequency. As another analytical step, Eq. (A5) can be replaced by a sum of sinusoids producing attenuated responses from several remote Doppler cells. The amplitude of the largest of these, normalized by $|C_{nk}|$, is a dimensionless ratio p which defines the PSLR as $-20 \log_{10}(p)$ or $-10 \log_{10}(p^2)$. The sum of squares of amplitudes in all remote Doppler cells, divided by $|C_{nk}|^2$, is a dimensionless ratio Q which defines the ISLR as $-10 \log_{10}(Q)$.

When ω_0 is small, Eq. (A7) is not valid and the correlator output contains extraneous responses from nearby, rather than remote, cells. The image is then smeared rather than speckled. Suppose that

$$\cos\omega_0 t \doteq 1 - \tfrac{1}{2}(\omega_0 t)^2, \text{ and } 0 \le t \le t_c \qquad \text{(A8)}$$

From Eqs. (A2) and (A6) this introduces a quadratic phase shift into the correlator, just as if an acceleration bias had been present as in Eq. (4). Superposition of a cubic term, Eqs. (17) and (18), will make the correlator output still more responsive to the contents of nearby Doppler regions, thus further smearing the image. There is no abrupt separation between acceptable and unacceptable amounts, but the limits imposed by Eqs. (5) and (18) are fairly representative.

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